A COUPLED MESHFREE/SCALED BOUNDARY METHOD

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Abstract The scaled boundary method is an excellent way to model unbounded domains. However, it is limited to linear problems. Many soft-ground geotechnical problems require both non-linear constitutive behaviour for the soil, to capture pre-failure deformations, and the presence of an unbounded domain. Adaptive meshfree methods are ideally suited to such problems. This paper couples a meshless local Petrov–Galerkin method for the near field with a meshless scaled boundary method of similar type for the far field. The method presented is novel as the degrees of freedom of all nodes in the support of the interface nodes are coupled to the stiffness of the unbounded domain, rather than just the nodes on the interface.

Keywords: meshfree method, scaled boundary method.

1. INTRODUCTION

While meshless methods are increasingly seen as future replacements for the conventional finite element method, they retain the shortcomings of the conventional finite element method when modelling singularities or infinite boundaries. Both features can, however, be dealt with efficiently using the Scaled Boundary Method (SBM) although this method cannot incorporate non-linear material behaviour unlike meshless methods. This paper describes the coupling of a meshless method to the SBM to yield a numerical method ideal for problems in areas such as geomechanics where both non-linear constitutive behaviour and accurate modelling of infinite boundaries are required.

In this study we use the Meshless Local Petrov–Galerkin (MLPG) method [1]. This method is based on a moving least squares (MLS) approximation

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for the displacement field, as are other popular meshless methods. The SBM is semi-analytical and was developed relatively recently by Wolf and Song [2, 3]. Understanding and interest in the SBM has since increased partly due to publication of a virtual work derivation of the method for elastostatics [4]. In its simplest form, a point in a domain is assigned a scaling centre, from which radiators are defined along which the displacement field is an analytical (i.e., closed-form) solution. In the circumferential direction the displacement field is approximated by conventional finite element type shape functions.

2. COUPLING THE METHODS

The coupled method is shown in Figure 1 for the case of a footing problem (later used to demonstrate the hybrid method). The MLPG method is used in the near field (i.e., close to an applied load or prescribed displacement condition). SBM elements are used along the material boundary of the meshless region, thus the remainder of the infinite domain is covered. Doherty and Deeks [5] have already combined the SBM with conventional finite elements. In that case coupling was relatively simple as the same shape functions are used in each method along the interface between the two zones. A first step in coupling the MLPG method and the SBM is to reformulate the SBM using the same shape functions in the circumferential direction as used in the MLPG method to produce a SBM without elements. This is also straightforward as

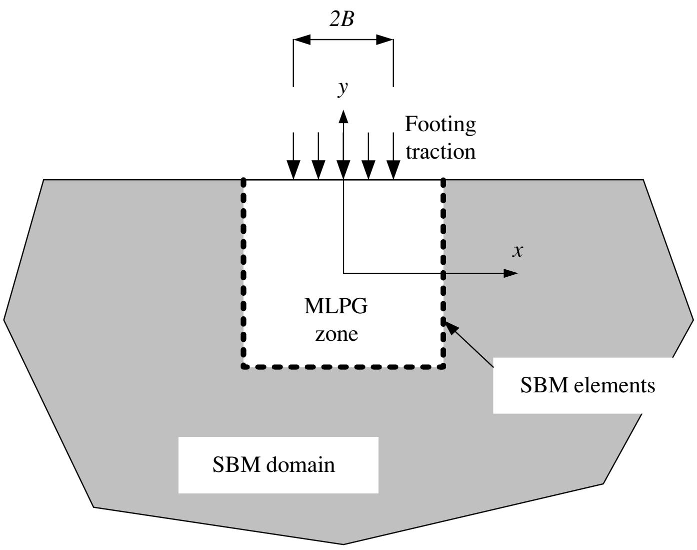

Figure 1. Hybrid method applied to the smooth footing problem.

the approximation is one-dimensional, along the boundary s. The procedure is described in detail in Deeks and Augarde [6].

Having produced an 'elementless' SBM it would seem to be straightforward to then couple MLPG with this method. However complications arise as the MLPG returns fictitious nodal values $\{\hat{u}_m\}$ to which the MLS approximation is fitted rather than actual nodal values, $\{u_u\}$ as returned by the SBM. This feature of meshless methods leads to difficulties in enforcement of essential boundary conditions, as highlighted elsewhere [7]. Along the SBM 'side' of the boundary the approximation is governed by nodes on the boundary. On the MLPG side, however, the approximation is influenced in addition by nodes inside the meshless domain. For these reasons, direct coupling is not possible and a penalty approach is used to couple the two methods. The SBM approximation is more restricted than the MLPG approximation to which it must be coupled, since at any point the latter is by definition based on fewer nodes. Therefore the MLPG approximation is restricted to the SBM values, rather than the other way round.

In brief, the coupling is implemented as follows. If Γ_i is the interface between the zone then equilibrium is satisfied in a weak sense in the meshless zone if

$$[K]\{\hat{u}_m\} - \int_{\Gamma_i} \left[N^2(x, y)\right]^T \{t\} d\Gamma_i = \{f\} \tag{1}$$

where $[K]$ is the stiffness matrix for the meshless region, $[N^2(x, y)]$ is a matrix containing meshless test functions, $\{t\}$ are the tractions along the interface between the zones and $\{f\}$ are the externally applied forces (assumed not to occur along the interface). A similar equilibrium expression can be written for the SBM zone in terms of both $\{u_u\}$ and $\{\hat{u}_m\}$, recalling that along the interface the two displacements are kept separate. Two further equations are obtained from enforcing compatibility along the interface. Derivation for the meshless zone begins from

$$[N^1(s)]\{u_u\} = [N^1(x, y)]\{\hat{u}_m\} \quad \text{over } \Gamma_i \tag{2}$$

where $[N^1(s)]$ is a matrix of shape functions for the SBM (note the single coordinate, s) and $[N^1(x, y)]$ is the corresponding matrix for the meshless region. Combining the equilibrium expression for each zone with its corresponding compatibility condition (the latter weighted with a penalty parameter $\alpha \gg 0$) leads to a system of $2(n_u + n_m)$ linear equations in $2(n_u + n_m)$ unknown nodal values (where n_u is the number of nodes on the interface and n_m is the total number of nodes in the meshless region). Following solution for displacements, stresses can be recovered in both zones. A full derivation is given elsewhere [8].

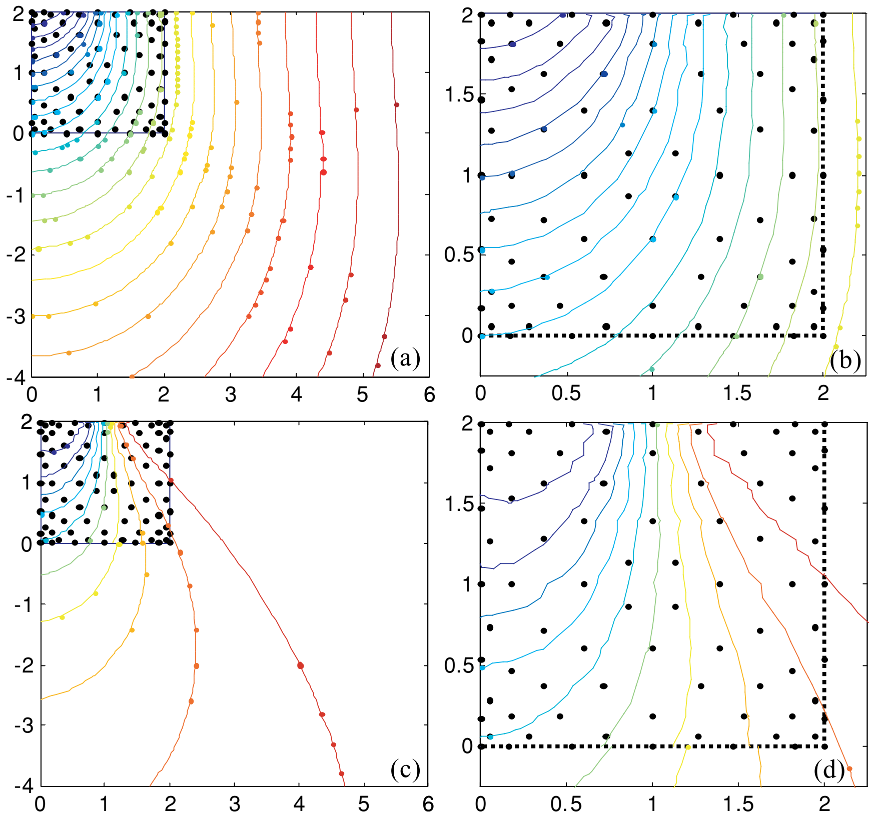

Figure 2. Results for the footing problem. Vertical displacements (a) and (b) in close up; vertical stresses (c) and (d) in close up.

3. AN EXAMPLE

The hybrid method outlined above is demonstrated on a simple elastostatics problem, namely a plane strain smooth footing of width 2 units on an elastic half-space (Figure 1). A uniform traction is applied in the negative y-direction over the footing width. Only one half of the problem is modelled due to symmetry; the symmetric boundary is the y-axis. A coarse irregular grid of 97 nodes covers the meshless region, which is square of size 2 units. (Grid spacing is based on Gauss–Lobatto intervals). Thirteen SBM nodes are used along the interface. Displacement and stress results are shown in Figure 2 for the meshless area (where the nodes are also shown) and in the infinite domain for ($0 \leq x \leq 6$; $-4 \leq y \leq 2$).

The plots show that the penalty method is successful in enforcing the coupling between the methods on the boundary as displacements are smooth across the interface. The resulting stress field derived from processing the meshless displacements and the SBM displacements separately is also seen to produce an acceptable result, although there are minor discrepancies between the zones visible along the interface which could be reduced by refinement along the interface.

4. CONCLUSIONS

A hybrid method has been described and demonstrated that couples a meshless method with the Scaled Boundary method thereby potentially allowing nonlinear behaviour in the near-field with infinite boundaries, and the economies that produces. Further work is in progress to test and extend this hybrid method particularly for use in geomechanics problems.

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